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ABSTRACT

The major intent of this project was to design microcomputer lessons utilizing the unique features of computers to teach a high school mathematics topic. After reviewing the literature, it was decided to write a series of three tutorial lessons for a class in high school trigonometry which would present the concepts of amplitude, period, phase shift, and vertical translation of a sine wave. Lessons were developed and preliminary trials of the materials were conducted using ten students from a trigonometry class and four students from a geometry class. These students' knowledge of the materials was tested before and after the use of the tutorial; additionally, each student completed a questionnaire giving reactions to the lessons. The computer lessons proved to need appropriate accompanying materials and instruction in order to be effective, but students seemed to enjoy the uniqueness of the computer presentation. It was concluded that future work on the project should include: (1) the development of printed materials to accompany the lessons; (2) improvement of some of the lessons, especially the legibility of the graphics of the third lesson; and (3) formal testing of the materials in high school classrooms. Program listings written in BASIC for the Compucolor II are included as well as 36 references. (Author/LLS)

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USE OF A MICROCOMPUTER IN THE TEACHING OF HIGH SCHOOL MATHEMATICS

A Project Report
Presented to
the Faculty of the Department of Mathematics
San Jose State University

In Partial Fulfillment
of the Requirements for the Degree
Master of Arts

by
Jean Bohrer

August 1981

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"PERMISSION TO REPRODUCE THIS MATERIAL HAS BEEN GRANTED BY
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J.B.

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CHAPTER 1

INTRODUCTION

Computers are appearing in many high schools today, partly due to the fact that they have become more affordable and more prevalent in society as a whole. It is the task of the mathematics teacher to determine how this learning tool can best be implemented into the classroom.

The trend in many of the commercially available programs seems to be to use the computer to introduce a topic and provide practice questions. Programs such as PILOT and DECAL are designed to help teachers develop lessons of this type. In research studies, the computer is often used to replace the textbook and merely serves as a disciplinary device to mete out information as the student progresses. However, when material is satisfactorily covered in the classroom with textbook and paper, then translating that same material onto the computer does not seem to contribute much to the lesson nor fully utilize the value of the computer. It is not sufficient to look through the curriculum and choose topics that can be "computerized". Instead, the computer should enhance or expand a particular topic.

Bell (1978) states that the computer should be used for subjects that currently are not being done very well by the teacher. Programs that are written for classroom use should take this idea into account. The computer can interact with each student and help recognize and drill weak areas. More significantly, the ability of the computer to do difficult computations and plot graphs quickly reduces the tedium that may be involved in a problem. This can open up many new topics that are not normally part of the curriculum and also provide many discovery-type experiences.

Fitting (1980) cited seven kinds of computer programs that can be used in computer assisted instruction. These programs can be drill, practice, tutorials, demonstrations, simulations, games, or problem solving. (For an explanation of each type, see Appendix A.1.) In the review of the computer literature that follows, some of these types of programs are utilized in teaching various topics in mathematics.

When I undertook this project I wanted to investigate the types of programs that were available to the high school mathematics teacher. I wished to examine the various ways that the computer was used in these programs. I sought an area of mathematics where very little had been written. I also looked for a topic that could be taught more effectively with the use of a computer.

After reviewing the literature, I decided to write a series of three tutorial lessons for a class in high school trigonometry which would present the concepts of amplitude, period, phase shift, and vertical translation of a sine wave. I developed the lessons and did some preliminary trials of the material using ten students from a trigonometry class and four students from a geometry class. I tested these students' knowledge of the material before and after the use of the tutorials and also had each student complete a questionnaire giving reactions to the lessons.

The computer lessons proved to need appropriate accompanying materials and instruction in order to be effective, but students seemed to enjoy the uniqueness of the computer presentation. Future work in this area should include several things. Development of printed materials to accompany the lessons would enable students to practice what they have learned. Improvement of some of the lessons, especially the legibility of the graphics in the third lesson, would help. Finally, the materials should be formally tested in high school classrooms.

CHAPTER 2

REVIEW OF LITERATURE

Section 1. General Math

For the pre-algebra student lacking in the fundamentals of mathematics, a tutorial type of learning environment can be very beneficial to the student. The computer has appeared in many math laboratories in order that the student may have individualized instruction, drill, and practice, which he might not receive in a large classroom situation.

For example, Kuehn and Modisett (1977) reported on the use of $MATH, a computational exercise program which the student works on for ten minutes per day. The difficulty of the problems is gradually increased, and records of the student's progress are kept. Wappingers Central School District (1978) instituted a remedial program with the use of a computer in a learning center. Besides providing practice and record keeping, this program can determine the best learning environment for a particular student through analysis of student responses. In a similar program, Project Master (1978) services eighteen students at a time but has a unique feature in that it takes into consideration career objectives. The student chooses a career option and

then is presented with problems that relate to that area.

In the area of general math, the programs used tend to concentrate on drill and practice for the student. While this provides immediate feedback, saves teacher time for grading, and is a service for the student, it does not fully utilize the unique features that a computer can provide.

Section 2. Algebra

In the field of algebra, the computer is used primarily for demonstration or problem solving. The computer can calculate and print data quickly. By examining this data, patterns can be seen and conjectures can be made.

For some problems a calculator is sufficient to perform the computational tasks. Olson (1979) found that with the use of a calculator, students could test out hypotheses by trying various numerical examples. For example, the students noted that $5^2 - 5 = 4^2 + 4$, $6^2 - 6 = 5^2 + 5$, and $1776^2 - 1776 = 1775^2 + 1775$. By using the calculator to test out similar examples, they conjectured that if $a-b=1$, then $a^2 - a = b^2 + b$. The hypothesis was then proved algebraically.

Snover and Spikell (1978) presented a problem involving more difficult computation in which the computer was used beneficially. They noted that $153 = 1^3 + 5^3 + 3^3$, i.e. that 153 is equal to the sum of the cubes of its digits. With the help of the computer, the student could search for other

numbers with this property. Once others are found, the student could compare the numbers and conjecture regarding the form these special numbers take.

The speed at which the computer can evaluate and plot functions is a great asset to the student. Morgan and Winzenread at California State University, Hayward have developed several programs that evaluate and plot functions, curve-fit functions to data which is entered, simulate the rolling of dice and compute the frequency distribution, and evaluate limits at a point by entering values closer and closer to that point. The computer is used during the class period to compute and show results immediately. This provides the student with immediate reinforcement and examples.

Shneiderman (1974) used the computer's plotting ability to investigate a question often overlooked in algebra due to the tedium of the task. The student is able to investigate the effect of changing the coefficients of a polynomial on the form that the graph takes. In a related exercise, the student may be given a graph of a polynomial and can then try different coefficients to find the equation which best fits the graph. By programming the computer to plot the function given the coefficients, the drudgery of the problem is eliminated.

Exercises in plotting functions can be made enjoyable with the help of the computer. Gabai (1976) uses graphcodes

to enable the student to relate the equation of a polynomial with its graph. Graphcodes involve forming letters and words by using equations defined over specific intervals. The student varies the form of the letters by varying the interval or coefficients of the equations used. The computer gives immediate printout of the graphs when equations are provided by the student.

The capacity of the computer to assist in discovery was used to an advantage by Zabinski and Fine (1979). The student examines the question regarding the kinds of roots (real, complex, multiple) that $ax^2 + bx + c = 0$ has when a,b, and c are chosen at random from a sepecified interval. The computer solves the equation and prints the frequency of the kinds of roots over a specified interval. The student uses this data to discover relationships and patterns that could not have been hypothesized before the computer was used. The problem is unusual and rarely examined, hence the student is "discovering" something new in mathematics.

Howard Jensen (1980) at Pioneer High School used the computer in his algebra classes to present and reinforce concepts covered during the regular class period. The students worked two at a time at the computer and interacted with it by reading the text and working on practice exercises. The topics of the programs included "Slope", "Distance",

"Midpoint", and "Equations of Lines", followed by a quiz. The computer was found to be a good motivational and reinforcement device.

Although the computer acts as a good motivational device, its presence in the curriculum should not be based solely on the fact that it provides a "change of pace" to the everyday classroom activities. The interactional and motivational benefits are just part of the instructional advantages that the computer offers. The opportunity for discovery by examining computer generated data should be recognized and implemented into classroom programs.

Section 3. Number Theory

There are several topics in number theory which can be examined more easily with the computational ability of the computer. Numerous articles have been written about Pascal's Triangle alone. Curley (1974) cites four different ways of generating Pascal's Triangle and provides BASIC and FORTRAN programs for one of the methods. When the student writes the program, he learns how the triangle is generated and then has a copy of it which he can use later. Hansen (1975) suggests a fifth way of generating Pascal's Triangle through the use of vectors.

Lund (1979) used the various printed forms of Pascal's Triangle to create an area of mathematics which gives

"hands on" experience. By using computer programs for the printing of the triangles, the students could create pictures. The students would experiment and refine their programs in order to produce the type of picture they desired. In this exercise the computer acted as demonstrator by showing the effect of various changes in the programs to produce various pictures.

Staib and Staib (1978) presented the often overlooked generalization of Pascal's Triangle to Pascal's Pyramid, a 3-dimensional array of numbers which gives trinomial coefficients. A BASIC program was written to give the trinomial coefficients for powers 0-15, and they also proved the validity of the construction of the pyramid.

The task of searching for prime numbers is a popular exercise in number theory. Bidwell and Fallon (1975) considered polynomial functions which generate primes over a specified interval. Finding such polynomials is a good problem for students to investigate. The computer can help generate polynomials and can quickly check which interval yields primes. Cohen (1976) considered the Mersenne primes, M_n, and wished to find for what n, M_n is prime. This can be solved via the Lucas-Lehmer sequence, $S_i \Big|_{i=1}^{\infty}$, where $S_1 = 4$, $S_{k+1} = S_k^2 - 2$ for $k = 1,2,3,\ldots$, and the theorem which says M_n is prime if and only if M_n divides S_{n-1}. The computer can be used to ease calculations by checking

if the conditions of the theorem are satisfied.

Other topics in number theory can be programmed in order to ease calculations in solving problems. Kappel (1976) used the computer and backtracking the Euclidean algorithm to find the numbers r and s such that $ra + sb = (a,b)$, the greatest common divisor of a and b. Hughes (1971) showed that by writing computer programs which would determine if a given polygon was constructible, the students were forced to clarify the conditions of Gauss' Existence Theorem for Constructible Regular Polygons. Also, in the process of making their programs more efficient, the students were introduced to other concepts regarding primes and divisibility.

Metz (1977) used the computer for trial and error computation in solving systems of equations involving the modulus. The equations were formulated by the students to answer the question of when runners would meet again on a track, given that they travel at different speeds. The concept of the modulus was brought into the problem in a natural way, and the computer gave immediate solutions to the conditions which needed to be satisfied.

McCabe and Ransom (1979) used the computer to "explore number patterns through a trial-and-error method". The students were presented with the question of whether or not there were numbers other than those of the form 2^n that

cannot be represented as the sum of more than two consecutive integers. The students wrote programs to confirm that only 2^n was of this type by showing how any other number could be written in the desired form. The programs solved the problem for any number entered.

In the field of number theory, the computer can be used to great advantage in discovering relationships. The speed of computation is one of the unique features of the computer and it can more effectively search for number patterns than can a teacher and a textbook. In most of the applications above, solving the problem using a computer requires that the students have a knowledge of programming. The solution to the problem is achieved through the writing of the program rather than the analysis of the program results. This makes the ability to program necessary for these lessons.

Section 4. Geometry

In geometry, where conjecture and proof is prevalent, the computer can assist the student in grasping this difficult area of high school mathematics. Muller (1977) presented an example that would be very difficult to investigate without the assistance of the computer. Muller wished to find the number of 60° and 120° triangles with integral side lengths. The computer was used to find some examples by searching within specified bounds. Relationships could be

explored among the data and conjectures would follow. Once this was done, formal proofs could be given. This process enables the student to see the origin of proof and better understand its purpose.

Another example involving conjecture was developed by Ouellette and Bennett (1979). The basic problem (with special cases used to help generalize) concerned n equally spaced points on a circle ($V_1, V_2, \ldots\ldots, V_n$) and a fixed point, P, also on the circle. The student was to conjecture regarding the value of the sum $PV_1^2 + PV_2^2 + \ldots\ldots + PV_n^2$. By using the computer to calculate various cases, conjectures could be made regarding the value of the sum and then a formal proof could be given.

Hay (1981) combined human reasoning with computer computation in order to discover the solution of a problem and its proof. The problem involved proving that a triangle was equilateral, and no direct method of proof was useful. After proving some supporting results, Hay used the computer to search for a counterexample. After analyzing the data, new relationships were discovered and a line of proof was suggested.

Flowcharts are fundamental to computer programs and the task of classifying geometric figures can be facilitated through the use of a flowchart and the computer. Dennis (1978) outlined a method which can be written for the

computer to classify any triangle or quadrilateral. It is the method which is most instructive to the student rather than the actual running of the program, which merely states the kind of figure when the vertices are entered.

In many of the computer applications, a student knowledge of programming is essential in order to get the maximum out of the experience. While the number of students who are able to program is increasing, there remain many students who cannot write programs. If the teacher writes lessons in which the learning takes place during programming, then this places a limit on the kind and number of students who can benefit from the lesson. In order to involve all students in discovery exercises using the computer, the teacher could creatively write programs which provide for discovery in the execution rather than in the writing of the programs.

Section 5. Advanced Topics

There have been some studies in which the computer was used throughout a calculus course as a supplement to the regular lecture. At Simmons College (1976) all calculus courses used canned computer programs. Canned programs take up less class time and homework time than student written programs since the student does not need to know how to write programs. The programs gave the students extra practice and

were written in such a way that the students felt as if they were in control of the calculations. As a consequence, students were more apt to believe such concepts as limits and convergence. Calculus seemed more concrete and they saw direct applications of it.

In another report by O'Loughlin (1976), the results of a study conducted with two calculus classes were discussed. One class was taught solely by teacher's lecture and the other used lecture and mini-computer, which provided practice and applications. It was found that the class which used the mini-computer performed as well on all concepts as the other class, but did significantly better on topics involving a function and its derivatives, the definite integral, and solving maximum-minimum problems. This study has shown that by using the mini-computer, the student can work on enrichment and topics not normally considered without weakening the topics usually covered.

Again, the ability of the computer to calculate quickly can be used when finding approximations to limits and integrals. Johnsonbaugh (1976) illustrated limits by using the computer to calculate partial sums of an infinite series. The printout of partial sums was then examined by the students to determine possible convergence or divergence. Flynn (1974) offers two methods which can be

programmed for the computer which will approximate a definite integral. The trapezoidal rule gives the student the feeling of what an integral is and how to approximate it. The Monte Carlo Method which uses random numbers generated by the computer is a good introduction to probability theory.

Baker (1977) offers a method other than using secants to find the derivative of a polynomial at a point, c. His method is to divide the polynomial by x-c and then divide the quotient by x-c. The remainder is the value of the derivative at x=c. The computer is used to do the computations and gives the derivative value directly to the student. Baker (1979) examines the curve $y = x\sin(1/x)$ and uses the computer and Newton's Method to find relative extrema. This enables the student to graph the curve and better understand its behavior over a specified interval.

When dealing with the subject of queuing theory, the computer can play an important role as simulator. Mathers (1976) uses the computer to simulate the operation of a one-chair barber shop. The computer works with random arrival times to determine such things as average waiting time, longest waiting time and longest line. The student can analyze the means and standard deviations in terms of the problem and suggest ways to alter the variables for other desired results.

The complex ideas in senior year mathematics can be made more concrete when the computer is used as a supplement to the course. An intuitive idea of limits, integrals, and derivatives can be achieved when using the computer for computations and tutorials. Simulation of experiments is fundamental in a probability and statistics course. The computer can be programmed to provide numerous situations which can then be examined by the students. This is an extremely effective use of the computer in the classroom.

CHAPTER 3

CHOOSING A TOPIC

After reviewing much of the literature on the use of the computer in the classroom, I became aware of a segment of the curriculum that was not very well represented. I found very few applications to trigonometry and in particular, the graphing of trigonometric functions. Shneiderman (1974) and Gabai (1976) wrote computer exercises to plot polynomial functions, and Jensen (1980) provided practice with linear equations. Trigonometric functions were not covered in the literature, and these functions are tedious to plot by hand.

It is within the subject of trigonometry and more specifically the graphing of trigonometric functions on which I chose to concentrate. Many examples of graphs need to be drawn in order to discover the effects of changing amplitude, period, phase shift, and vertical translation. The computer can evaluate and plot points quickly so that many examples can be generated in a small amount of time. This allows the student to concentrate his efforts on analysis and conjecture.

In the area of using the computer as a tutorial device, I was influenced by my participation in the PLATO project at the University of Illinois (1974-76). As a student,

I was presented with material relating to physics and chemistry through the computer. Although some simulation programs were given, most lessons consisted of drill and practice. This became boring very quickly since the level of student interaction with the computer was minimal. I wanted my programs to be more interactive and more discovery-oriented than the ones I experienced on PLATO.

My desire to utilize the graphics capabilities of the computer was influenced by two sets of programs which I have seen. The first was written by Howard Jensen at Pioneer High School on an Apple II computer. The graphics that he used involved the drawing of axes and plotting points and lines to illustrate the concepts of slopes, distances, and equations of lines. His programs presented new material and provided problems for practice. The second set of programs was written by Professor Marjorie Fitting of San Jose State University on a Compucolor II computer. Two of these programs dealt with characteristics of the graphs of sine waves and a demonstration of graphs chosen by the student.

Although Jensen used some graphics, the programs he wrote did not satisfy one important criterion: the computer should be used to cover a topic which the teacher cannot do very well in the classroom with only a blackboard and an overhead projector. I feel Jensen's topics are all

covered satisfactorily without the computer. The graphics used in Professor Fitting's programs, however, are difficult for the teacher to do by hand. These programs utilize the capabilities of the computer although they are difficult to do without any previous orientation to the topic.

I chose to write programs dealing with the topic of characteristics of the sine wave. I wanted to design the programs so that they could be used to introduce the topic as well as to reinforce the topic after a presentation in class. I also wanted the graphics to be the most prominent aspect of each program. With the extensive use of graphics, I wanted to create a discovery-oriented lesson in which the student feels that he is in control. Lastly, the student will not require a knowledge of programming in order to participate in the lesson.

There are a number of ways in which my programs differ from those by Fitting on the same subject. In my programs, the pace is slower and there is more repetition of graphs to illustrate the effect of each constant. Each of the four constants is treated separately and then all four are put together into the general equation of the sine wave. This slower pace and separation of topics should make it possible for these programs to be used to introduce the topic. Many leading questions are also included in the programs in order to help the student discover the effect of changing constants.

I wrote my programs in BASIC for the Compucolor II because I felt that this computer has one of the best graphics capabilities of any of the small computers. Different foreground and background colors can be chosen which makes it possible to put many graphs on the same display. Text can be written in two sizes of type and can be placed anywhere on the screen along with the graphics. This allows questions to be asked while the graphics are displayed and permits the labelling of graphs.

I have written a discovery-oriented lesson utilizing the graphics capabilities of the computer that demonstrates the characteristics of the sine wave. I hope that the lesson will provide the student with an experience that the teacher, working without a computer, is unable to match. By using student volunteers to run the programs, I hope to solicit their suggestions and record their progress with the topic. The results of the evaluation will tell me if the purpose of my programs has been fulfilled. Students should recognize that the ability to choose and graph many equations is the primary feature of the lesson. I wish to determine if my programs can be understood by students working alone. Through student responses I also want to know how these programs could be integrated into a particular course.

CHAPTER 4

DESCRIPTION OF PROGRAMS

To cover the topic of characteristics of the sine wave, I wrote three programs. The first introduces the concepts of amplitude and period. The second deals with phase shift and vertical translation with respect to the x-axis. The third program combines all four constants into the general equation of a sine wave. All programs demonstrate these concepts through graphs. Textual material used in these programs came in part from Discovering Second Year Algebra and Trigonometry by Roger A. Johnson and Modern Trigonometry by William Wooton, Edwin F. Beckenbach, and Mary P. Dolciani.

The following is a list of the objectives for the three computer programs.

The student should be able to . . . . . .

1. locate the maximum and minimum of a sine function.
2. locate the x-intercepts of a sine wave.
3. state in his own words the effect of multiplication by A in Y = A*SIN(X), related to the graph of Y = SIN(X).
4. define amplitude.
5. state whether or not two sine waves have the same amplitude.
6. locate the y-intercept of a sine wave.

7. state whether or not two sine waves have the same y-intercept.

8. state the interval in which a sine curve pattern repeats itself (integer multiples of pi only).

9. state the effect that multiplication by B in Y = SIN(B*.) has on the graph of Y = SIN(X).

10. state whether or not two sine waves have the same period (interval of repetition).

11. define period.

12. state the difference between changing amplitude and changing period of a graph.

13. state what effect the addition of C in Y = SIN(X + C) has on the graph of Y = SIN(X).

14. define phase shift.

15. state what effect the addition of D in Y = SIN(X) + D has on the graph of Y = SIN(X).

16. state the difference between phase shift and vertical translation of a graph.

17. using the equation Y = A * SIN(B(X + C)) + D, state the change that each constant makes on the graph of Y = SIN(X).

18. within certain number value constraints, choose an equation to graph and state the four quantities (amplitude, period, phase shift, and vertical translation) relating to the graph chosen.

In order to achieve these objectives, a variety of learning principles and computer features are included in the programs. One common technique in teaching is the use of repetition. There are several places in the programs where repetition is used. The drawing of axes and the graph of Y = SIN(X) is repeated for every set of graphs drawn.

For each of the four constants in the general equation of a sine wave, the student has the opportunity to pick values three times and see three graphs. Textual material is also repeated in order to reinforce concepts covered earlier in the programs. Previously defined terms such as amplitude and period are used in later questions in order that the student becomes comfortable with these words.

To maintain student interest, a variety of types of questions were included in the programs. This is difficult to do since open ended questions are not practical on a computer. The questions have to require only a single, short response. My questions are of three types: "Yes or No", multiple choice, and single number answers. The placement of these questions on the same page as a graphics display, and placement without the benefit of a graphics display also adds to the variety.

It is very easy to use color with the Compucolor II computer, since changing color requires only the pressing of two keys. Color can be used to great advantage in teaching provided it is used in a consistent manner and with discretion. In my programs, textual material and questions are always written in white, answers and explanations to the questions are always in yellow. Important words are written in a contrasting color to the rest of the text. This use of color helps the student to

follow the lesson more easily and to recognize what kind of material is coming up. Color is also used in the sine graphs. The value that a student chooses for a constant is given a color and the resulting equation and graph is drawn in the same color. This helps the student to locate the graph which matches the value he chooses, and to refer to previously drawn graphs. Color can help the student feel secure when following a program. Too much color or color used inconsistently only confuses the student.

The manner in which material is arranged on the screen also determines how well the student can follow the program. When questions or text is presented, some built-in delays are included to give the student time to read and think. An entire display of text is never given to the student all at once. This makes the student feel less rushed and therefore more inclined to read the text carefully. On the graphics displays, new text is placed starting at the top of the screen and moving vertically down. When many questions need to be asked, some of the previously placed text is erased to avoid cluttering the display. When all work on a particular graphics display is finished, the directions to continue are placed in a flashing white box in order for the student to notice them on a filled screen. The arrangement of material in delayed portions and following vertically contributes to the security of the student just

as does the proper use of color.

Whether teaching with or without a computer, student interaction is a vital part of the learning process. Even though a computer is a machine and lacks the personal touch that a teacher provides, it has the advantage of providing a one-to-one teaching situation. I have incorporated into my programs several features which put the student in control.

The first way in which the student can interact with the computer is by setting the pace. Although there are some preprogrammed delays, the student can further control the pace through such statements as "press return to continue". The individual can then view a display as long as necessary. Related to this is the opportunity for repetition. Besides the repetition that I have written in, the student is asked at the end of each program if he would like to repeat it. A student may require more than one exposure to the lesson and in my programs he has control over this.

Besides control over the mechanics of running the programs, the student can actually take part in making the displays. He is offered a choice of numbers with which to make graphs and then the computer demonstrates the particular concept by using the student chosen values. The student is discovering through graphs made from his

choices. The ability he has of altering the pictures gives him the feeling of control even though the computer does the evaluating and plotting for him.

The last feature which contributes to student interaction lies in the questions and answers. Whenever a question is asked and the student provides an answer, the computer responds to his answer. He is told whether he is right or wrong and is also given a reason why. This allows the student to examine a wrong answer and compare it to the correct one in order to discover his mistake. Information that the student provides when answering a question or choosing data is not ignored by the computer. He is instead given immediate feedback, something very helpful to the learning process and difficult for the teacher to provide for each student.

It is important that these three programs deal with a topic that the computer seems to handle uniquely well. I feel that the use of color, choice of constants, and ease of graphing which the computer contributes to the lesson makes this topic successful. It is not feasible for the teacher to cover this topic to the same extent as can be accomplished with the computer.

In terms of writing the programs, a few problems were encountered regarding the graphics displays. When making the axes using the graphics mode, color bleeding results.

This means that when sine waves are graphed, the section of the axes intercepted by the sine wave changes color to that of the sine wave. With four sine waves graphed on the same set of axes, the color bleeding is distracting and makes for less accurate graphs. This problem was solved by plotting the axes in the character mode. Not only do the axes lines become thinner, but portions are erased when sine graphs are plotted over them. This eliminates color bleeding in the vicinity of the axes. Graph-to-graph color bleeding still occurs but the incidence of it is minimal and also inevitable with this computer.

Once I changed to graphing the axes in the character mode, I encountered another problem in aligning the axes unit markings with the sine graphs. Since every character block is equal to a 2 x 4 graphics block, axes should be marked in some multiple of four on the y-axis and a multiple of two on the x-axis. Unfortunately, I had already placed text on certain lines which limited the axes divisions I could use. Since I required y-axis unit markings in every fifteenth space and x-axis unit markings in every twenty-fifth space, I solved the problem by using different vertical or horizontal line characters which placed the line markings at various positions in the character block.

CHAPTER 5

USING THE PROGRAMS

Fourteen San Jose State students ran the programs and responded to them. Ten of the students were enrolled in a trigonometry course. Four students were from an intuitive geometry class and had not taken any trigonometry. All students were volunteers.

In order to obtain some evaluation of the effectiveness of the programs, several materials were written. For all students, verbal and written instructions in operating the computer terminal were giren. (See Appendix A.2.) For the geometry students, an additional written introduction to the sine wave was provided since they lacked the necessary background. (See Appendix A.3.) A pretest on content was given to all the trigonometry students on the day that volunteers were obtained. (See Appendix A.4.) The results of the eight questions on the pretest were used to determine whether the students were seeing the material for the first time. Geometry students were given the pretest immediately before running the programs. The pretest covered several objectives of the computer lesson. For all students, a post-test was given after they completed the computer lesson. (See Appendix A.5.) The results of the eight questions on

the post-test were used to determine whether the students were brought up to mastery of the content by use of the programs. Most students spent approximately thirty minutes running the programs. The post-test covered the same objectives but consisted of different problems with the exception of one question that appeared on both the pretest and the post-test.

After the completion of the post-test, a questionnaire was filled out by each student. (See Appendix A.6.) The questionnaire asked the student to choose his preferred method of learning and the recommended time to use the programs. Each student also rated various aspects of the lesson on a scale from one to five. Three open-ended questions asked for likes, dislikes, and suggestions for improvement of the lesson.

Several problems were encountered which affected the interpretation of the results. On the day that I gave the pretest to the trigonometry class, the instructor had covered the material earlier in the period. Student performance on the pretest was, therefore, very good. This eliminated the possibility of my programs being the first exposure to the topic. It is for this reason that I chose to test four geometry students. I wanted to determine the effectiveness of the programs when students had no previous orientation to the topic.

I was also informed by a trigonometry student that in class a different form of the general equation of the sine wave was used. The class used Y = A * SIN(B * X + C) + D, and I used Y = A * SIN(B(X + C)) + D in my programs. The form used affects the manner in which the phase shift is recognized. This difference in equations and terminology caused confusion in some students while running the programs.

The testing of the geometry students brought additional difficulties. A total unfamiliarity with the sine function made it difficult to run the programs in a small amount of time. Two of the students were willing to spend additional time in repeating the programs and subsequently scored well on the post-test. The other two students were not motivated by the topic and found it difficult. One student spent only ten minutes on the programs.

Of the ten trigonometry students, eight scored the same or better on the post-test than on the pretest. The other two students scored seven out of eight on the pretest and six out of eight on the post-test. It seems that student success with this topic is due partly to the teacher's lecture and partly to the reinforcement provided by the computer lesson. With this group of students the analysis of the pretests and post-tests are not very informative since it is difficult to determine which method was responsible for their success. The student questionnaires,

therefore, deserve closer attention.

When the students were asked for their preferred method of learning, one person chose reading a textbook, one chose a tutor, and one person preferred a lecture or a tutor. Seven students preferred a computer lesson. This is a surprising number. However, these students volunteered to participate in this lesson and were probably more interested in computers than the other members of their class. Their choice does reflect a positive response to the lesson.

In rating aspects of the lesson itself, seven students felt the pace was "just right", two thought it was a little slow, and one person thought it was slightly fast. Eight students thought that the questions and instructions were very clear or mostly clear. Two students felt that they were not clear enough. Six students felt that the lessons thoroughly covered the subject, and four students felt it was close to thorough coverage. Nine students were satisfied with the amount of material on the screen, and one student felt it was a little cluttered. For the overall quality of the lesson, seven ranked it as excellent, two ranked it as good and one person ranked it as fair.

When the students were asked to recommend all ways in which this lesson could be used, the response was somewhat evenly divided. Four people thought it could be used as a test. Five students felt that the lessons could be the

initial presentation of the topic. Eight people thought it would make a good review in a later course, and nine people recommended that it be used after a lecture on the topic.

Although I wrote the programs so that they could be used to introduce the topic, only half of the students recommended that the programs be used in that way. Nearly all of the students suggested that they be used after the teacher has lectured on the topic. This response tells me that the computer is a worthwhile supplement to a lesson but it still cannot compete with a teacher in providing the entire lesson. Students still prefer various degrees of interaction with the teacher.

Most students commented that the graphs were what they liked most about the lesson. Many appreciated the ability to see their choices quickly on the computer. Student reaction supported my contention that my programs use the computer to present graphs that cannot be drawn efficiently in class. Other students remarked that they liked having the opportunity to repeat graphs and proceed at their own pace. Pacing and repetition were two of the features that I wanted my programs to contain.

Four students said that the part of the lesson they liked the least was that the graphs were sometimes cluttered in the third program. This could be solved by clearing

the screen and putting the one graph which is used to answer the subsequent questions on a new set of axes. One student did not like the fact that if he was left alone, there would be no one to answer a question if he was confused. This comment again points to the need for human interaction. The teacher should provide sufficient orientation to the lesson, and an assistant on hand would be helpful when the computer is being used.

Regarding suggestions for improvement of the lesson, most students offered programming suggestions such as reverse imaging, time limits on answers, sound stimulus, increasing the range of the graphs, and erasing information. These suggestions reflect the familiarity that many students already have with computers. In terms of the material presented, one student wanted more of an explanation regarding the calculation of the period from the equation. This can be done without the use of a computer and therefore I did not include it in the lesson. However, a supplementary program on this topic might make the presentation more complete.

Through the participation of the four geometry students, I was provided with results from students with no background on the topic. None of the students could answer any questions on the pretest. Results on the post-test were varied. One student did not attempt to answer any questions and another

student scored one right. The two students who spent the most time running my programs performed the best on the post-test. One student scored four out of eight questions correct and the other had a perfect score. This showed me that my programs are effective in introducing the topic provided that the student is motivated to spend the time running and repeating the programs. I feel that the motivation required to do this would be present in a trigonometry student since the lesson pertains to material with which he is somewhat familiar and he would be held responsible for the topic covered by the programs. The geometry students had no such additional motivation.

Questionnaire results were similar to the trigonometry group results for the learning method preferred and the recommendation for use of the lesson. However, the rating of the lesson was markedly different. All aspects of the lesson received low ratings, with the exception of amount of material on the screen. This is probably due to the fact that the subject matter was unfamiliar to the students. Without the benefit of trigonometry background, the lesson seemed too fast, unclear, and incomplete to the student. The use of this lesson at the proper time in a trigonometry class would eliminate much of this confusion.

CHAPTER 6

CONCLUSION

The computer lesson was designed to teach a topic in which the unique features of a computer could be used to do something that is difficult for a teacher to do alone. My programs did this through the use of computer graphics, and the majority of students recognized this as being an important feature of the lesson.

Regarding the ability of the students to understand the lesson, I can recommend some steps which might improve this. The presence of the teacher or an aid to answer questions while students run the programs is necessary. I also feel that the most understandable and effective programs for a class are those written by the teacher of that class. This assures students of seeing similar terminology and style and would eliminate some of the confusion that might occur in the execution of the programs. If the teacher uses programs written by someone else, they should be carefully screened for any notational differences.

Because of the difficulties in motivation of the geometry students, I feel that these programs should be used primarily in a trigonometry course. Although only half of the students recommended that the programs be used to

introduce the topic, I feel that the discovery experience which this lesson offers is important. I feel that this lesson can be the initial presentation of the topic if some changes are made. The students who participated in this experiment worked through all three programs in one sitting. If I were to use these in class to introduce the topic, I would give the students only one program per day. This would slow down the pace enough to provide the time to "discover" the material and also to discuss each idea before the students continue.

Now that I have an idea of how these programs can best be used, it remains for them to be tested. They should be given to a trigonometry class after sufficient background material has been covered but before the characteristics of sine waves is discussed. Under these conditions the results of a pretest and post-test would be meaningful.

In order for the entire lesson to be more effective, materials to be used by the teacher in class could be developed. Students want some teacher interaction and the coordination of in-class lecture and computer lesson is an important area for future study. A brief orientation to the topic and to the computer is necessary before running the computer program. Follow-up materials in class are very important. Some method of testing the student's grasp of the material in the computer lesson could be devised.

Related subject matter which can be done effectively in class could also be presented after the computer lesson. In short, the development of unit materials that use the computer in part would be a worthwhile project.

Besides the graphing of trigonometric functions, there are other topics in mathematics that can be taught more effectively if the computer were used. For example, a similar lesson involving discovery through graphing could be written for quadratic relations. When studying probability, the computer could be used to simulate experiments. The data supplied can then be used by the student to find the probability of events using the relative frequency. The computer may be used to improve the teaching of domains and ranges of functions, or to draw 3-dimensional figures for use in geometry. Any of these topics could provide the basis for future work in the area of computer assisted instruction. The unique features of the computer make many areas of mathematical study more accessible and meaningful than a teacher could working without the computer.

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APPENDIX A.1

KINDS OF COMPUTER PROGRAMS FOR INSTRUCTIONAL PURPOSES
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Computer managed instruction (CMI) is an instructional management system involving organizing curricula and student data, monitoring student progress, diagnosis and prescription, evaluation of learning outcomes and provision of planning information for instructors.

Computer assisted instruction (CAI) is a teaching process which can involve one or more of the following:

1. Drill: repetition of facts, definitions, theorems, formula which are to be committed to memory.

2. Practice: application of previously learned principles or algorithms to sample situations.

3. Tutorial: presentation of new knowledge combined with evaluations of how well student is acquiring the knowledge. The evaluation is used to route the student through a branching program to provide review, practice or drill, and reinforcement. In one form the tutorial is a program in which the student will read some material and answer a question (or two) about the reading; the computer will record the number of answers correct and incorrect. In more imaginative forms the tutorial helps the student to discover the principles to be learned and provides adequate application of the principles.

4. Demonstration: demonstration of cause-effect relationships involving student input of independent variables and immediate display of results. In a demonstration each independent variable usually has an effect on one unique characteristic of the result.

5. Simulations: presentation of a problem oriented environment in which the student makes irrevocable decisions and suffers the consequences. In a simulation the sequence of activities is linear and each activity + decision + result affects the possibilities available in the next activity.

6. Games: drill and practice using skill and/or strategy. Student may compete or cooperate with another student, with the computer, or for a score or other result indicating his level of achievement.

7. Problem Solving: computational procedures which allow students to gather data or operate on data the results of which will lead them to the solution of the problem at hand.

Both CMI and CAI involve use of existing programs for instructional purposes. In many instructional settings students are involved in writing their own programs for problem solving or for experience when learning how to program. This is not considered as CAI or CAI.

APPENDIX A.2

VERBAL AND WRITTEN INSTRUCTIONS BEFORE THE LESSON

VERBAL:

1. The keyboard is similar to a typewriter except that letters are in capitals, so the shift key is not needed to make capitals.
2. You must use the shift key and the " [" / 2] " key to make quotation marks.
3. You must use the " [Ø] " key for zero.
4. The " [= / -] " key will be used to make a negative sign.
5. Whenever you answer a question or enter a number you must hit the RETURN key afterwards.

WRITTEN:

1. To start the first program type LOAD "SIN1" and hit the RETURN key. After the noise stops, type RUN and hit the RETURN key. Now read and follow the instructions on the screen.

   Note: { Whenever you answer a question or enter data, you must hit the RETURN key afterwards.
   If you make a typing error hit the [←] key to back up over it.

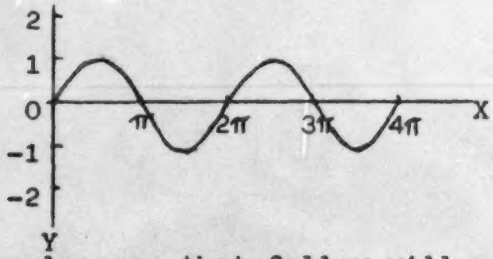
2. After the first program is finished, type LOAD "2SIN" and hit the RETURN key. Then type RUN and hit the RETURN key. This will start the second program. Follow the lesson.
3. After the second program is finished, type LOAD "SIN3" and hit the RETURN key. Then type RUN and hit the RETURN key. This will start the third program. Follow the lesson.
4. After all 3 programs have been run, you will take a post-test and fill out a questionnaire.

APPENDIX A.3

INTRODUCTION TO THE SINE WAVE

In nature, there are many phenomena that can be described using a wave graph. Ocean waves and sound waves are two examples. These waves are called <u>sine waves</u> and have the following shape:

The mathematical equation which describes these waves is called the <u>sine function</u>. The most basic sine function is $Y = SIN(X)$, which is graphed on the axes below. The symbol "π" is the irrational number "pi", which is approximately 3.14.



$Y = SIN(X)$

The lessons that follow will mathematically examine the graphs of sine functions.

APPENDIX A.4

PRETESTS

Pretest NAME: BRIAN HAGER 46

① the general equation of a sine function is $Y = A \cdot \sin(B(X+C)) + D$ where A, B, C & D are constants (real numbers) Match the constant with the quantity that it affects from the list at right.

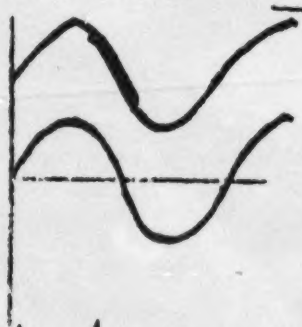
A affects AMPLITUDE
B affects PERIOD
C affects PHASE SHIFT
D affects VERTICAL TRANSLATION

- Period
- Phase Shift
- Amplitude
- Vertical Translation

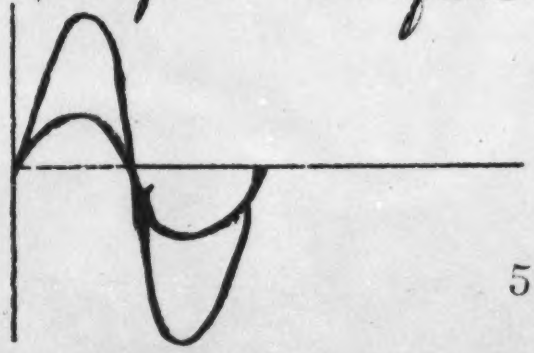
② The sine wave equation $Y = 4 \cdot \sin(3X) + 1$ has amplitude 4.

③ The sine wave equation $Y = \frac{1}{2}\sin(2(X+1)) - 3$ has phase shift -3.

④ Which of the four quantities in #1 has been changed in the graph of the two functions below? VERTICAL TRANSLATION



⑤ Which of the four quantities in #1 has been changed in the graph of the two functions below? AMPLITUDE



APPENDIX A.5

POST-TESTS

Postest

NAME: BRIAN HAGER 61

① The general equation of a sine function is $Y = A \cdot \sin(B(X+C)) + D$ where A, B, C & D are constants (real numbers). Match the constant with the quantity that it affects from the list at right.

A affects AMPLITUDE
B affects PERIOD
C affects PHASE SHIFT
D affects VERTICAL TRANSLATION

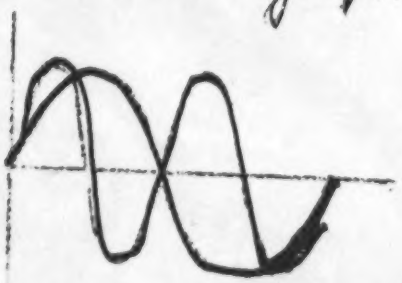
{ Period
Phase Shift
Amplitude
Vertical Translation

② The sine wave equations $Y = 3 \cdot \sin(2X) - 1$ has amplitude 3

③ The sine wave equations $Y = 2\sin(3(X+4)) + 6$ has phase shift 4

● Which of the four quantities in #1 has been changed in the graph of the two functions below?

~~Phase Shift~~



⑤ Which of the four quantities in #1 has been changed in the graph of the two functions below?

~~Period~~

APPENDIX A.6

QUESTIONNAIRES

Questionnaire:

① What method do you prefer when learning?

2) ☐ lecture
3) ☐ reading a textbook
4) ☐ group discussion
5) ☐ using an individual tutor
1) ☒ computer lessons

② Rate the following aspects of the computer lesson which you just completed. (Circle the number)

(a) pace of the lesson	too slow 1	2	just right (3)	4	too fast 5
(b) clarity of questions/ instructions	Not Clear 1	(2)	3	4	Very Clear 5
(c) subject matter coverage	Incomplete 1	2	3	4	Thorough (5)
(d) amount of material on screen	Too cluttered 1	2	Satisfactory (3)	4	Not Enough 5
(e) Overall quality of lesson	Poor 1	2	3	4	Excellent (5)

③ When would you recommend using this computer lesson? (Check all that apply)

☐ as the initial presentation of this topic
☒ after a lecture on this topic.
☐ as a test on this topic
☐ as a review in a later course.

(Turn over)

④ What did you like most about the lesson?

It's an exciting way to learn. The machine motivates the student to continue. A book can be tossed on to a shelf & ignored. The computer keeps my attention span & interest going much longer.

⑤ What did you like least about the lesson?

⑥ Please make some suggestions for improvement of this lesson.

MAKE IT AVAILABLE FOR CALCULUS in the fall!

Thank you!

A.7.1 Program "SIN1"

```
5 PLOT 12,23
8 PLOT 14
10 PRINT "SINCE SINE IS A FUNCTION, IT CAN BE GRAPHED"
12 PRINT "ON A SET OF COORDINATE AXES."
15 PRINT  : PRINT
20 PRINT "THE X AXIS WILL BE MARKED IN UNITS OF PI,"
22 PRINT "AND THE Y AXIS IN ONE UNIT INCREMENTS."
25 PRINT  : PRINT  : PRINT
30 PRINT "THE FIRST SINE FUNCTION THAT WILL BE GRAPHED"
32 PRINT "IS Y=SIN(X) WHERE X IS IN RADIANS."
33 PRINT  : PRINT
35 INPUT "PRESS RETURN TO SEE Y=SIN(X) GRAPHED.";A
38 PLOT 15
40 PLOT 12,20
50 GOSUB 4000
70 PLOT 3,4,0
80 INPUT "WHAT IS THE MAXIMUM OF THIS FUNCTION?";A
90 IF A=1 THEN 120
100 PLOT 3,4,1
110 PRINT "SORRY, THE MAXIMUM IS 1."
115 GOTO 130
120 PLOT 3,4,1
125 PRINT "RIGHT!"
130 PLOT 3,4,2
140 INPUT "WHAT IS THE MINIMUM OF THIS FUNCTION?";B
150 IF B=-1 THEN 190
160 PLOT 3,4,3
170 PRINT "SORRY, THE MINIMUM IS -1."
180 GOTO 210
190 PLOT 3,4,3
200 PRINT "GOOD JOB!"
210 PLOT 3,4,4
220 INPUT "PRESS RETURN TO CONTINUE.";A
225 REM  ERASE TEXT ON TOP OF PAGE
230 FOR X=0 TO 5
240 PLOT 3,4,X
250 PRINT "                                                  "
260 NEXT X
280 PLOT 23
282 PLOT 3,4,0
285 PRINT "NOW LET'S SEE THE EFFECT ON THE GRAPH     "
290 PLOT 3,4,1
295 PRINT "WHEN WE LET |A BE A NUMBER OTHER THAN 1"
296 PLOT 3,4,2
298 PRINT "IN THE EQUATION |Y=A*SIN(X)."
300 PLOT 3,0,26
310 PRINT "CHOOSE A VALUE FOR |A SUCH THAT -3<=|A<=3 AND YOU"
BREAK
READY
```

```
312 PRINT "WILL SEE THE NEW GRAPH."
320 PLOT 22
330 INPUT "A= ";A
335 IF A>3 OR A<-3 THEN 344
340 GOSUB 3000
342 GOTO 350
344 PLOT 3,6,28
346 PRINT "SORRY,";A;" IS OUT OF RANGE"
350 PLOT 19
355 PLOT 3,0,29
360 INPUT "TRY ANOTHER VALUE FOR A. A= ";A
361 IF A>3 OR A<-3 THEN 374
370 GOSUB 3000
371 GOTO 380
374 PLOT 3,31,29
375 PRINT "OUT OF RANGE"
380 PLOT 21
385 PLOT 3,0,30
390 INPUT "TRY ONE MORE VALUE FOR A. A= ";A
392 IF A>3 OR A<-3 THEN 404
400 GOSUB 3000
402 GOTO 410
404 PLOT 3,32,30
406 PRINT "OUT OF RANGE"
410 GOSUB 6000
510 PLOT 12
520 PRINT "FOR ALL THE GRAPHS YOU MADE, DID THE X "
522 INPUT "INTERCEPTS REMAIN THE SAME?(Y/N)";A$
526 PLOT 19
530 IF A$="Y" THEN 560
540 PRINT TAB(4);"WRONG. ALL GRAPHS CROSSED THE X AXIS"
545 PRINT TAB(4);"AT INTERVALS OF PI."
550 GOTO 570
560 PRINT TAB(4);"RIGHT! ALL GRAPHS CROSSED THE X AXIS"
561 PRINT TAB(4);"AT INTERVALS OF PI."
570 PRINT  : PRINT
580 PLOT 23
590 PRINT "BY CHOOSING DIFFERENT VALUES FOR |A DID THE"
591 INPUT "MAXIMUM AND MINIMUM OF THE CURVES CHANGE?(Y/N)";B$
594 PLOT 19
595 IF B$="Y" THEN 605
600 PRINT TAB(4);"SORRY, CHANGING Y VALUES MADE TALLER OR"
601 PRINT TAB(4);"SHORTER GRAPHS, AFFECTING THE MAX AND MIN."
603 GOTO 608
605 PRINT TAB(4);"RIGHT! CHANGING Y VALUES MADE TALLER OR"
606 PRINT TAB(4);"SHORTER GRAPHS."
608 PRINT  : PRINT  : PRINT
609 FOR K=1 TO 2000 : NEXT K
610 PRINT "THIS VERTICAL CHANGE AFFECTS WHAT IS CALLED THE"
620 PRINT "AMPLITUDE OF THE CURVE."
BREAK
READY
```

```
625 PRINT
627 PRINT TAB(10);"THE AMPLITUDE OF THE CURVE IS DEFINED"
628 PRINT TAB(10);"AS HALF THE VERTICAL DISTANCE FROM THE"
629 PRINT TAB(10);"MAXIMUM TO THE MINIMUM POINT."
630 PLOT 2,18,66,242,97,66,97,48,18,48,18,66,255
640 FOR K=1 TO 4000 : NEXT K
646 FOR X=0 TO 63 STEP 2
647 PLOT 3,X,21
648 PRINT "c"
649 NEXT X
650 PLOT 3,0,24
660 PRINT "ANOTHER WAY WE CAN VARY THE GRAPH OF Y=SIN(X)"
670 PRINT "IS BY MULTIPLYING X BY A NUMBER."
690 PRINT  : PRINT
700 PRINT "WE WILL AGAIN GRAPH Y=SIN(X) AND SEE THE EFFECT OF"
710 PRINT "INSERTING A VALUE FOR |B IN THE EQUATION |Y=SIN(B*X)."
730 INPUT "PRESS RETURN TO CONTINUE";A
732 PLOT 12,20
735 GOSUB 4000
740 PLOT 23
741 PLOT 3,4,0
742 PRINT "HOW MANY INTERVALS OF PI DOES IT TAKE"
743 PLOT 3,4,1
744 INPUT "FOR THE CURVE TO REPEAT ITSELF?";A
748 IF A=2 THEN 753
749 PLOT 3,4,2
750 PRINT "SORRY, THE INTERVAL IS 2*PI."
751 GOTO 758
753 PLOT 3,4,2
755 PRINT "CORRECT!"
758 PLOT 3,0,26
760 PRINT "CHOOSE A VALUE FOR |B SUCH THAT 0<|B<=3 AND"
762 PRINT "|Y=SIN(B*X) WILL BE GRAPHED FOR YOU."
763 PLOT 22
769 PLOT 3,0,28
770 INPUT "B= ";B
772 IF B<=0 OR B>3 THEN 784
780 GOSUB 5000
782 GOTO 790
784 PLOT 3,7,28
786 PRINT "SORRY,";B;" IS OUT OF RANGE"
790 PLOT 19
795 PLOT 3,0,29
800 INPUT "TRY ANOTHER VALUE FOR B. B= ";B
805 IF B<=0 OR B>3 THEN 814
810 GOSUB 5000
812 GOTO 820
814 PLOT 3,32,29
816 PRINT "OUT OF RANGE"
820 PLOT 21
825 PLOT 3,0,30
830 INPUT "CHOOSE 1 LAST VALUE FOR B. B= ";B
835 IF B<=0 OR B>3 THEN 844
BREAK
READY
```

```
840 GOSUB 5000
842 GOTO 860
844 PLOT 3,34,30
846 PRINT "OUT OF RANGE"
860 GOSUB 6000
950 PLOT 12
960 PRINT "FOR ALL THE GRAPHS YOU JUST MADE, WERE THE MAXIMUMS"
970 INPUT "AND MINIMUMS THE SAME?(Y/N)";A$
980 IF A$="Y" THEN 1020
990 PRINT TAB(4);"SORRY, THEY ALL HAD A MAXIMUM OF 1 AND A"
1000 PRINT TAB(4);"MINIMUM OF -1."
1010 GOTO 1040
1020 PRINT TAB(4);"CORRECT! THEY ALL HAD A MAXIMUM OF 1"
1030 PRINT TAB(4);"AND A MINIMUM OF -1."
1040 PRINT
1050 PRINT "HENCE, THEIR AMPLITUDES WERE THE SAME."
1060 PRINT  : PRINT
1065 FOR K=1 TO 2000 : NEXT K
1070 PRINT "AS YOU CHANGED THE VALUE OF |B IN THE EQUATION"
1080 PRINT "|Y=SIN(B*X), DID THE INTERVAL WITH WHICH THE CURVE"
1090 PRINT "REPEATED ITSELF REMAIN THE SAME FOR ALL GRAPHS?"
1095 INPUT "(Y/N)";B$
1100 IF B$="N" THEN 1150
1110 PRINT TAB(4);"SORRY,AS |B CHANGED, THE GRAPH WAS EITHER"
1120 PRINT TAB(4);"HORIZONTALLY COMPRESSED OR STRETCHED, WHICH"
1130 PRINT TAB(4);"CHANGED THE INTERVAL OF REPETITION."
1135 PRINT  : PRINT
1140 GOTO 1190
1150 PRINT TAB(4);"RIGHT! AS |B CHANGED, THE GRAPH WAS FITHER"
1160 PRINT TAB(4);"HORIZONTALLY COMPRESSED OR STRETCHED OUT."
1180 PRINT  : PRINT  : PRINT
1190 FOR K=1 TO 3000 : NEXT K
1200 PRINT  : PRINT
1210 PRINT "WHEREAS AMPLITUDE DESCRIBES |VERTICAL COMPRESSION"
1220 PRINT "AND EXPANSION, THE TERM PERIOD IS USED TO DESCRIBE"
1230 PRINT "|HORIZONTAL COMPRESSION AND EXPANSION."
1240 PRINT
1250 PRINT TAB(10);"IF P IS A NUMBER ON THE X AXIS SUCH THAT"
1260 PRINT TAB(10);"THE CURVE BETWEEN THE INTERVAL FROM P TO"
1270 PRINT TAB(10);"2P IS A REPETITION OF THE CURVE BETWEEN"
1280 PRINT TAB(10);"THE INTERVAL FROM 0 TO P, THEN P IS THE"
1290 PRINT TAB(10);"PERIOD OF THE CURVE."
1300 PLOT 2,18,37,242,102,37,102,14,18,14,18,37,255
1305 PRINT
1310 PRINT "WOULD YOU LIKE TO RUN THIS PROGRAM AGAIN?"
1320 INPUT "(Y/N)";A$
1330 IF A$="Y" THEN 5
1340 END
2998 REM  SUB 3000 PLOTS A*SIN(X)
3000 FOR X=0 TO 100 STEP 2
3010 PLOT 2,X+7,15*A*SIN(X/8)+70,255
3020 NEXT X
3030 PLOT 3,48,2
BREAK
READY
```

```
3035 PLOT 14
3040 PRINT "Y=";A;"*SIN(X)  "
3045 PLOT 15
3060 RETURN
3998 REM  SUB 4000 PLOTS AXES
4000 FOR X=3 TO 54
4005 PLOT 3,X,14
4010 PRINT "f"
4015 NEXT X
4020 PLOT 3,16,13
4025 PRINT "m"
4030 PLOT 3,28,13
4035 PRINT "o"
4040 PLOT 3,41,13
4045 PRINT "m"
4046 PLOT 3,53,13
4047 PRINT "o"
4050 FOR X=2 TO 25
4055 PLOT 3,3,X
4060 PRINT "o"
4065 NEXT X
4070 PLOT 3,1,3
4075 PRINT "3e"
4080 PLOT 3,1,6
4085 PRINT "2 "
4090 PLOT 3,1,10
4095 PRINT "1i"
4100 PLOT 3,1,14
4105 PRINT "0f"
4110 PLOT 3,0,18
4115 PRINT "-1e"
4120 PLOT 3,0,21
4125 PRINT "-2 "
4130 PLOT 3,0,25
4135 PRINT "-3i"
4140 PLOT 3,55,14
4145 PRINT "(UNITS OF"
4150 PLOT 3,55,15
4155 PRINT "    PI)"
4157 PLOT 17
4160 FOR X=0 TO 100 STEP 2
4165 PLOT 2,X+7,15*SIN(X/8)+70,255
4170 NEXT X
4175 PLOT 3,48,0
4180 PRINT "Y=SIN(X)"
4185 RETURN
4998 REM  SUB 5000 PLOTS SIN(B*X)
5000 FOR X=0 TO 100 STEP 2
5010 PLOT 2,X+7,15*SIN(X/8*B)+70,255
5020 NEXT X
5030 PLOT 3,48,2
BREAK
READY
```

```
5035 PLOT 14
5040 PRINT "Y=SIN(";B;"*X)  "
5045 PLOT 15
5050 RETURN
5998 REM  SUB 6000 SIGNALS TO CONTINUE
6000 PLOT 3,50,26
6005 PLOT 23
6010 PRINT "EXAMINE THESE"
6020 PLOT 3,50,27
6030 PRINT "GRAPHS, THEN"
6040 PLOT 3,50,28
6050 PRINT "PRESS RETURN"
6060 PLOT 3,50,29
6070 PRINT "TO CONTINUE"
6071 PLOT 31
6073 PLOT 2,98,27,242,127,27,127,4,98,4,98,27,255
6074 PLOT 15
6080 INPUT A
6090 RETURN
READY
```

```
10 PLOT 12,23
30 PRINT "IN THE LAST EXERCISE YOU FOUND THAT AMPLITUDE  OF A"
40 PRINT "GRAPH COULD BE VARIED BY CHANGING THE VALUE OF |A IN"
50 PRINT "THE EQUATION |Y=A*SIN(X)."
60 PRINT  : PRINT
70 PRINT "YOU ALSO DISCOVERED THAT A GRAPH CAN BE HORIZONTALLY"
80 PRINT "COMPRESSED OR EXPANDED, WHICH CAUSES A CHANGE IN THE"
90 PRINT "PERIOD. THIS WAS DONE BY CHANGING THE VALUE OF |B IN "
100 PRINT "THE EQUATION |Y=SIN(B*X)."
102 PRINT  : PRINT
105 INPUT "PRESS RETURN TO CONTINUE.";A
110 PRINT  : PRINT  : PRINT  : PRINT  : PRINT
120 PRINT "NOW WE'D LIKE TO SEE HOW THE ADDITION OF A NUMBER"
130 PRINT "TO THE EQUATION Y=SIN(X) AFFECTS THE GRAPH."
150 PRINT
160 PRINT "WE WILL START WITH THE GRAPH OF Y=SIN(X) AND THEN"
170 PRINT "CHOOSE DIFFERENT VALUES FOR |C IN THE EQUATION"
180 PRINT "|Y=SIN(X+C)."
185 PRINT  : PRINT  : PRINT
190 INPUT "PRESS RETURN TO CONTINUE.";A
195 PLOT 12
200 GOSUB 4000
210 PLOT 3,4,0
215 PLOT 23
220 INPUT "WHAT IS THE Y-INTERCEPT OF THIS GRAPH?";A
230 IF A=0 THEN 270
240 PLOT 3,4,1
250 PRINT "NO, THE CURVE CROSSES THE Y-AXIS AT 0."
260 GOTO 290
270 PLOT 3,4,1
280 PRINT "CORRECT!"
290 PLOT 3,0,26
300 PRINT "CHOOSE SOME VALUES FOR |C IN THE EQN. |Y=SIN(X+C)"
310 PRINT "TO SEE HOW THIS AFFECTS THE GRAPH. LET -3<=|C<=3."
320 PLOT 22
330 INPUT "C= ";C
335 IF C>3 OR C<-3 THEN 344
340 GOSUB 3000
342 GOTO 350
344 PLOT 3,7,28
346 PRINT "SORRY,";C;" IS OUT OF RANGE"
350 PLOT 3,0,29
360 PLOT 19
370 INPUT "TRY ANOTHER VALUE FOR C. C= ";C
375 IF C>3 OR C<-3 THEN 384
380 GOSUB 3000
382 GOTO 390
384 PLOT 3,32,29
386 PRINT "OUT OF RANGE"
BREAK
READY
```

```
390 PLOT 3,0,30
400 PLOT 21
405 IF C>3 OR C<-3 THEN
410 INPUT "CHOOSE 1 LAST VALUE FOR C. C= ";C
415 IF C>3 OR C<-3 THEN 424
420 GOSUB 3000
422 GOTO 430
424 PLOT 3,34,30
426 PRINT "OUT OF RANGE"
430 GOSUB 6000
440 INPUT A
450 PLOT 12,23
460 PRINT "FOR ALL THE GRAPHS YOU MADE, WHICH OF THE FOLLOWING"
470 PRINT "IS CORRECT?"
480 PRINT TAB(10);"(1) THEY HAD THE SAME AMPLITUDE."
490 PRINT TAB(10);"(2) THEY HAD THE SAME PERIOD."
500 PRINT TAB(10);"(3) BOTH (1) AND (2)."
510 PRINT TAB(10);"(4) NEITHER (1) NOR (2)."
520 PRINT
525 INPUT "ENTER THE NUMBER OF YOUR RESPONSE.";A
530 IF A=3 THEN 580
540 PRINT TAB(4);"SORRY, THE CORRECT RESPONSE IS 3. ALL GRAPHS"
550 PRINT TAB(4);"HAD AN AMPLITUDE OF 1 AND A PERIOD OF 2*PI."
570 GOTO 600
580 PRINT TAB(4);"GOOD JOB! ALL GRAPHS HAD AN AMPLITUDE OF 1"
590 PRINT TAB(4);"AND A PERIOD OF 2*PI."
600 PRINT
610 INPUT "DID ALL GRAPHS HAVE THE SAME Y-INTERCEPT?(Y/N)";B$
620 IF B$="N" THEN 660
630 PRINT TAB(4);"WRONG. THE Y-INTERCEPTS CHANGED AS THE"
640 PRINT TAB(4);"CURVES SHIFTED LEFT OR RIGHT."
650 GOTO 670
660 PRINT TAB(4);"RIGHT! THE Y-INTERCEPTS CHANGED AS THE"
665 PRINT TAB(4);"GRAPH SHIFTED LEFT OR RIGHT."
670 PRINT  : PRINT
680 FOR K=1 TO 1000 : NEXT K
690 PRINT "THE AMOUNT OF HORIZONTAL TRANSLATION OF A GRAPH IS"
700 PRINT "CALLED THE PHASE SHIFT."
705 PRINT  : PRINT
710 PRINT TAB(10);"IN THE EQUATION |Y=SIN(X+C), THE CONSTANT"
720 PRINT TAB(10);"|C IS CALLED THE PHASE SHIFT OF THE GRAPH."
730 PLOT 2,18,50,242,102,50,102,38,18,38,18,50,255
740 PRINT
750 FOR K=1 TO 3000 : NEXT K
760 FOR X=0 TO 63 STEP 2
765 PLOT 3,X,23
770 PRINT "c"
775 NEXT X
780 PRINT  : PRINT
BREAK
READY
```

```
790 PRINT "LET US NOW INVESTIGATE WHAT CHANGES OCCUR WHEN A"
800 PRINT "NUMBER IS ADDED TO SIN(X)."
810 PRINT "Y=SIN(X) WILL BE PLOTTED AGAIN, AND YOU WILL THEN"
820 PRINT "CHOOSE VALUES FOR |D IN THE EQUATION |Y=SIN(X)+D."
825 PRINT
830 INPUT "PRESS RETURN TO CONTINUE.";A
840 PLOT 12
850 GOSUB 4000
860 PLOT 23
870 PLOT 3,4,0
880 PRINT "CHOOSE VALUES FOR |D IN THE EQUATION"
885 PLOT 3,4,1
890 PRINT "|Y=SIN(X)+D SUCH THAT -2<=|D<=2."
900 PLOT 3,0,26
910 PLOT 22
920 INPUT "D= ";D
925 IF D>2 OR D<-2 THEN 934
930 GOSUB 5000
932 GOTO 940
934 PLOT 3,7,26
936 PRINT "SORRY,";D;" IS OUT OF RANGE"
940 PLOT 3,0,27
950 PLOT 19
960 INPUT "TRY ANOTHER VALUE FOR D. D= ";D
965 IF D>2 OR D<-2 THEN 974
970 GOSUB 5000
972 GOTO 980
974 PLOT 3,32,27
976 PRINT "OUT OF RANGE"
980 PLOT 3,0,28
990 PLOT 21
1000 INPUT "CHOOSE 1 LAST VALUE FOR D. D= ";D
1005 IF D>2 OR D<-2 THEN 1014
1010 GOSUB 5000
1012 GOTO 1020
1014 PLOT 3,34,28
1016 PRINT "OUT OF RANGE"
1020 GOSUB 6000
1030 INPUT A
1040 PLOT 12,23
1050 PRINT "FOR THE GRAPHS YOU JUST MADE, WERE THE AMPLITUDES"
1060 INPUT "THE SAME?(Y/N)";A$
1070 IF A$="Y" THEN 1100
1080 PRINT TAB(4);"WRONG. ALL GRAPHS HAD AN AMPLITUDE OF 1."
1090 GOTO 1110
1100 PRINT TAB(4);"GOOD! ALL GRAPHS HAD AMPLITUDE 1."
1110 PRINT  : PRINT
1120 PRINT "DID THE POSITIONS OF THE MAXIMUMS AND MINIMUMS"
1130 INPUT "CHANGE AS |D WAS CHANGED?(Y/N)";B$
BREAK
READY
```

```
1140 IF B$="Y" THEN 1170
1150 PRINT TAB(4);"SORRY, THE MAXIMUMS AND MINIMUMS MOVED"
1155 PRINT TAB(4);"UP OR DOWN AS THE VALUE OF |D CHANGED."
1158 PRINT  : PRINT  : PRINT
1160 GOTO 1182
1170 PRINT TAB(4);"RIGHT! THEY MOVED UP OR DOWN."
1180 PRINT  : PRINT  : PRINT  : PRINT
1182 FOR X=1 TO 1500 : NEXT X
1190 PRINT "WHEREAS CHANGING THE VALUE OF |C IN |Y=SIN(X+C)"
1200 PRINT "CAUSES |HORIZONTAL TRANSLATION OF A GRAPH (OR"
1210 PRINT "PHASE SHIFT),CHANGING THE VALUE OF |D IN |Y=SIN(X)+D"
1220 PRINT "CAUSES |VERTICAL TRANSLATION OF A GRAPH."
1230 PRINT  : PRINT
1240 PRINT TAB(10);"IN THE EQUATION |Y=SIN(X)+D, THE CONSTANT"
1250 PRINT TAB(10);"|D IS CALLED THE VERTICAL TRANSLATION OF"
1260 PRINT TAB(10);"THE GRAPH WITH RESPECT TO THE X-AXIS."
1270 PLOT 2,18,58,242,102,58,102,42,18,42,18,58,255
1280 PLOT 3,0,29
1290 INPUT "WOULD YOU LIKE TO RUN THIS PROGRAM AGAIN?(Y/N)";A$
1300 IF A$="Y" THEN 10
1310 END
3000 REM  PLOTS GRAPH OF Y=SIN(X+C)
3010 FOR X=0 TO 100 STEP 2
3020 PLOT 2,X+7,15*SIN(X/8+C)+70,255
3030 NEXT X
3040 PLOT 3,48,2
3050 PLOT 14
3060 PRINT "Y=SIN(X+";C;")  "
3070 PLOT 15
3080 RETURN
4000 REM  PLOTS AXES
4005 FOR X=3 TO 54
4010 PLOT 3,X,14
4015 PRINT "f"
4020 NEXT X
4025 PLOT 3,16,13
4030 PRINT "m"
4035 PLOT 3,28,13
4040 PRINT "o"
4045 PLOT 3,41,13
4050 PRINT "m"
4055 PLOT 3,53,13
4060 PRINT "o"
4065 FOR X=2 TO 25
4070 PLOT 3,3,X
4075 PRINT "o"
4080 NEXT X
4085 PLOT 3,1,3
4090 PRINT "3e"
4095 PLOT 3,1,6
BREAK
READY
```

```
4100 PRINT "2 "
4105 PLOT 3,1,10
4110 PRINT "1i"
4115 PLOT 3,1,14
4120 PRINT "0f"
4125 PLOT 3,0,18
4130 PRINT "-1e"
4135 PLOT 3,0,21
4140 PRINT "-2 "
4145 PLOT 3,0,25
4150 PRINT "-3i"
4155 PLOT 3,55,14
4160 PRINT "(UNITS OF"
4165 PLOT 3,58,15
4168 PRINT "PI)"
4170 PLOT 17
4171 FOR X=0 TO 100 STEP 2
4172 PLOT 2,X+7,15*SIN(X/8)+70,255
4173 NEXT X
4175 PLOT 3,48,0
4180 PRINT "Y=SIN(X)"
4185 RETURN
5000 REM  PLOTS Y=SIN(X)+D
5010 FOR X=0 TO 100 STEP 2
5020 PLOT 2,X+7,15*SIN(X/8)+70+15*D,255
5030 NEXT X
5040 PLOT 3,48,2
5050 PLOT 14
5060 PRINT "Y=SIN(X)+";D;"  "
5070 PLOT 15
5080 RETURN
6000 REM  SIGNAL TO CONTINUE
6005 PLOT 3,50,26
6010 PLOT 23
6020 PRINT "EXAMINE THESE"
6030 PLOT 3,50,27
6040 PRINT "GRAPHS, THEN"
6050 PLOT 3,50,28
6060 PRINT "PRESS RETURN"
6070 PLOT 3,50,29
6080 PRINT "TO CONTINUE."
6081 PLOT 31
6082 PLOT 2,98,27,242,127,27,127,4,98,4,98,27,255
6083 PLOT 15
6090 RETURN
READY
```

```
10 PLOT 12,23
20 PRINT "YOU ARE NOW AWARE OF 4 CONSTANTS THAT AFFECT THE"
30 PRINT "FORM OF THE GRAPH Y=SIN(X).  THIS ENABLES US TO"
40 PRINT "STATE THE GENERAL EQUATION OF A SINE WAVE:"
45 PRINT  : PRINT
50 PLOT 14
60 PRINT TAB(10);"Y = |A * SIN( B * ( X + C ) ) + D"
70 PLOT 15
80 PRINT  : PRINT
90 PRINT "CHOOSE THE NUMBER OF THE CORRECT RESPONSE TO THE"
100 PRINT "QUESTIONS THAT FOLLOW."
105 PRINT
110 PRINT TAB(6);"(1)  PERIOD"
120 PRINT TAB(6);"(2)  VERTICAL TRANSLATION OF GRAPH"
130 PRINT TAB(6);"(3)  PHASE SHIFT"
140 PRINT TAB(6);"(4)  AMPLITUDE"
150 PRINT  : PRINT
160 INPUT "WHAT QUANTITY DOES |A AFFECT?";A
170 IF A=4 THEN 200
180 PRINT TAB(4);"NO. |A AFFECTS THE AMPLITUDE OF THE CURVE."
190 GOTO 210
200 PRINT TAB(4);"CORRECT!"
210 PRINT
220 INPUT "WHAT QUANTITY DOES B AFFECT?";B
230 IF B=1 THEN 260
240 PRINT TAB(4);"SORRY, B AFFECTS THE PERIOD OF THE CURVE."
250 GOTO 270
260 PRINT TAB(4);"GOOD!"
270 PRINT
280 INPUT "WHAT QUANTITY DOES C AFFECT?";C
290 IF C=3 THEN 320
300 PRINT TAB(4);"WRONG. C TELLS YOU THE PHASE SHIFT."
310 GOTO 330
320 PRINT TAB(4);"RIGHT!"
330 PRINT
340 INPUT "WHAT QUANTITY DOES D AFFECT?";D
350 IF D=2 THEN 380
360 PRINT TAB(4);"NO. D AFFECTS THE VERTICAL TRANSLATION."
370 GOTO 385
380 PRINT TAB(4);"GOOD JOB!"
385 PLOT 3,35,30
390 INPUT "PRESS RETURN TO CONTINUE";A
395 PLOT 12
396 PLOT 3,0,10
400 PRINT "SOME OR ALL OF THESE CONSTANTS MAY APPEAR IN THE"
410 PRINT "EQUATION OF A PARTICULAR SINE WAVE."
420 PRINT "IN THE EXERCISE THAT FOLLOWS, Y=SIN(X) WILL BE"
430 PRINT "GRAPHED.  YOU WILL THEN CHOOSE VALUES FOR |A, B,"
440 PRINT "C AND D ONE AT A TIME TO CHANGE THE GRAPH."
BREAK
READY
```

```
445 PLOT 3,35,30
450 INPUT "PRESS RETURN TO CONTINUE.";A
470 PLOT 12
480 GOSUB 4000
490 PLOT 3,4,0
494 A=0
495 B=1
496 C=0
497 D=0
500 INPUT "LET |A BE |1, |1.5, OR |2.  |A=?";A
510 PLOT 22
520 PLOT 3,39,0
530 PRINT "Y=";A;"*SIN(X)"
540 GOSUB 5000
545 PLOT 3,4,1
550 INPUT "LET B BE .5, 1, OR 2.  B=?";B
560 PLOT 19
570 PLOT 3,39,1
580 PRINT "Y=";A;"*SIN(";B;"*X)"
590 GOSUB 5000
595 PLOT 3,4,2
600 INPUT "LET C BE 0, 1, OR 2.  C=?";C
610 PLOT 21
620 PLOT 3,39,2
630 PRINT "Y=";A;"*SIN(";B;"*(X+";C;"))"
640 GOSUB 5000
650 PLOT 3,4,3
660 INPUT "LET D BE -1, 0, OR .5.  D=?";D
670 PLOT 18
680 PLOT 3,39,3
690 PRINT "Y=";A;"*SIN(";B;"*(X+";C;"))+";D;
700 GOSUB 5000
710 PLOT 3,0,26
715 PLOT 23
720 PRINT "EXAMINE THE LAST GRAPH AND EQUATION."
725 PRINT
730 INPUT "WHAT IS THE AMPLITUDE?";M
740 IF M=ABS(A) THEN PRINT "RIGHT!" : GOTO 770
750 PRINT "SORRY, THE AMPLITUDE IS ";ABS(A)
770 GOSUB 6000
810 INPUT "THE PERIOD IS HOW MANY INTERVALS OF PI?";N
820 IF N=2/ABS(B) THEN PRINT "CORRECT!" : GOTO 840
830 PRINT "SORRY, THE PERIOD IS ";2/ABS(B);" INTERVALS OF PI."
840 GOSUB 6000
850 INPUT "WHAT IS THE PHASE SHIFT?";P
860 IF P=C THEN PRINT "GOOD!" : GOTO 880
BREAK
READY
```

```
870 PRINT "NO, THE PHASE SHIFT IS";C
880 GOSUB 6000
890 INPUT "WHAT IS THE DISPLACEMENT FROM THE X-AXIS?";Q
900 IF Q=D THEN PRINT "YOU'RE RIGHT!" : GOTO 920
910 PRINT "NO, THE DISPLACEMENT IS";D
920 PRINT
930 INPUT "DO YOU WANT TO TRY ANOTHER SET OF GRAPHS?(Y/N)";A$
940 IF A$="Y" THEN 470
950 PLOT 12,23
960 PRINT "WE CAN NOW SUMMARIZE WHAT WE KNOW RELATING THE"
970 PRINT "EQUATIONS AND GRAPHS OF SINE WAVES."
980 PRINT  : PRINT
990 PLOT 14
1000 PRINT TAB(10);"Y = |A * SIN( B * ( X + C ) ) + D "
1010 PRINT  : PRINT
1020 PRINT TAB(12);"|nAn IS THE AMPLITUDE"
1025 PRINT
1030 PRINT TAB(12);"2*PI/nBn IS THE PERIOD"
1035 PRINT
1040 PRINT TAB(12);"C IS THE PHASE SHIFT"
1045 PRINT
1050 PRINT TAB(12);"D IS THE VERTICAL TRANSLATION"
1060 PLOT 15
1070 PRINT  : PRINT  : PRINT
1080 INPUT "WOULD YOU LIKE TO RUN THIS PROGRAM AGAIN?(Y/N)";A$
1090 IF A$="Y" THEN 10
1100 END
4000 REM  PLOTS AXES
4005 PLOT 20
4010 FOR X=3 TO 54
4015 PLOT 3,X,14
4020 PRINT "f"
4025 NEXT X
4030 PLOT 3,16,13
4035 PRINT "m"
4040 PLOT 3,28,13
4045 PRINT "o"
4050 PLOT 3,41,13
4055 PRINT "m"
4060 PLOT 3,53,13
4065 PRINT "o"
4070 FOR X=2 TO 25
4075 PLOT 3,3,X
4080 PRINT "o"
4085 NEXT X
4090 PLOT 3,1,3
4095 PRINT "3e"
4100 PLOT 3,1,6
4105 PRINT "2_"
4110 PLOT 3,1,10
4115 PRINT "1i"
4120 PLOT 3,1,14
BREAK
READY
```

```
4125 PRINT "0f"
4130 PLOT 3,0,18
4135 PRINT "-1e"
4140 PLOT 3,0,21
4145 PRINT "-2 "
4150 PLOT 3,0,25
4155 PRINT "-3i"
4160 PLOT 3,55,14
4165 PRINT "(UNITS OF"
4170 PLOT 3,58,15
4175 PRINT "PI)"
4180 PLOT 17
4185 FOR X=0 TO 100 STEP 2
4190 PLOT 2,X+7,15*SIN(X/8)+70,255
4195 NEXT X
4200 PLOT 3,46,28
4210 PRINT "Y=|A*SIN(B*(X+C))+D"
4220 RETURN
5000 REM  GRAPHS Y=ASIN(BX+C)+D
5010 FOR X=0 TO 100 STEP 2
5020 PLOT 2,X+7,A*15*SIN(B*(X/8+C))+70+D*15,255
5030 NEXT X
5040 RETURN
6000 REM  ERASES LINES
6010 FOR K=1 TO 2000 : NEXT K
6020 PLOT 3,0,28
6030 PRINT "                                                    "
6040 PRINT "                                                    "
6050 PLOT 3,0,28
6060 RETURN
READY
```